1) The compression ratio of a diesel engine is 15 to 1; that is, air in a cylinder is compressed to 1/15 of its initial volume. Use the values $C_V = 20.8 \text{ J/mol} \cdot \text{K}$ and $\gamma = 1.40$ for air.
   a) If the initial pressure is $1.0 \times 10^5 \text{ Pa}$ and the initial temperature is $27^\circ \text{C}$, find the final pressure and the temperature after adiabatic compression.
   b) How much work does the gas do during the compression if the initial volume of the cylinder is $1.0 \times 10^{-3} \text{ m}^3$?
      
      a) In an adiabatic process: $P_i V_i^\gamma = P_f V_f^\gamma$ and $\frac{V_f}{V_i} = \left( \frac{P_i}{P_f} \right)^{\frac{1}{\gamma - 1}} \Rightarrow P_f = P_i \left( \frac{V_i}{V_f} \right)^{\gamma - 1} = 45 \times 10^5 \text{ Pa}.$
      
      \[ \frac{T_i V_i^{\gamma - 1}}{T_f V_f^{\gamma - 1}} = \frac{P_i V_i}{P_f V_f} \Rightarrow T_f = T_i \left( \frac{V_i}{V_f} \right)^{\gamma - 1} = 290 \text{ K}. \]
      
      b) For an adiabatic process $W = -\Delta U = \Delta \left( n C_V \left( T_i - T_f \right) \right) = \left( \frac{P_i V_i}{R T_i} \right) C_V \left( T_i - T_f \right) = -0.49 \text{ kJ}$.

2) The graph in fig. 1 shows a pV-diagram of the air in a human lung when a person is inhaling and then exhaling a deep breath. Such graphs, obtained in clinical practice, are normally somewhat curved, but we have modeled one as a set of straight lines of the same general shape. (The pressure shown is the gauge pressure, not the absolute pressure.)

   a) How many joules of net work does this person’s lung do during one complete breath?
      The pressure change is due to changes in the amount of gas in the lung, not to temperature changes. Think of your own breathing. Your lungs do not expand because they’ve gotten hot.
   b) If the temperature of the air in the lung remains a reasonable $20^\circ \text{C}$, what is the maximum number of moles in this person’s lung, $n_{\text{max}}$, during a breath? Which point in the graph corresponds to this $n_{\text{max}}$?
   c) If the temperature only changes from $20^\circ \text{C}$ to $37^\circ \text{C}$ estimate the change in $n_{\text{max}}$ in part ‘b’.

![Figure 1: pV-diagram.](image)

   a) $W$ done in a cycle is equal to the area captured in pV-diagram;
      each square is $1 \text{ mm Hg} \cdot 0.1 L = \frac{1.0 \times 10^5 \text{ Pa} \cdot 0.1 \times 10^{-3} \text{ m}^3}{760} = 13 \times 10^3 \text{ J}$
      \[ \Rightarrow W = 1.0 \text{ J} > 0 \text{ as the cycle goes clockwise.} \]
   b) When inhaling finished, at point a in the diagram, $n_a = n_{\text{max}} = \frac{P_a V_a}{R T_a} = 0.06 \text{ mol.}.$
   c) Using $T = 310 \text{ K}$ instead of $T = 293$ would not change the $n_{\text{max}} = \frac{P V}{R T_{\text{max}}}$. point, and one can still estimate using point a.

   If $n$ is changing, then we need to know the processes in one other diagram; either one of $V T, p T, n T, n p, n V$ would work. In part b we easily said $T$ = const.
3) The graph in fig. 2 shows a pV-diagram for $n = 3.25$ mol of ideal helium, He, gas. Part ca of this process is isothermal.

\[
\text{monatomic ideal gas} \quad C_p = \frac{5}{2} R, \quad C_V = \frac{3}{2} R
\]

a) Find the pressure of the He at point a.
b) Find the temperature of the He at points a, b, and c.
c) How much heat entered or left the He during segments ab, bc, and ca? In each segment, did the heat enter or leave?
d) By how much did the internal energy of the He change from a to b, from b to c, and from c to a? Indicate whether this energy increased or decreased.

Figure 2: pV-diagram for He gas.

\[ p \times 10^5 (\text{Pa}) \]
\[ V (\text{m}^3) \]
\[ P_a - P_b = ? \]
\[ T_a = T_b = ? \]
\[ 2.0 \]
\[ 0 \quad 0.010 \quad 0.040 \]

\[ a, b, c \] point in PV = nRT

\[ P_e = P_b = 2 \times 10^5 \text{ Pa} \quad V_b = V_c = 0.040 \text{ m}^3 \quad T_a = T_c = 296 \text{ K} = T_a \]

\[ P_a = \frac{nRT_a}{V_a} = 8.0 \times 10^5 \text{ Pa} = P_b \]

\[ T_b = \frac{P_b V_b}{nR} = 1184 \text{ K}. \]

\( n = 3.25 \) and const.

before doing these kind of problems it is a good practice to write down quick info about pVT for each point from the diagram, specify the processes are iso-something.

all info in the diagram:

\[ P_a = P_b \quad \text{(i)} \]
\[ P_e = 2 \times 10^5 \text{ Pa} \]
\[ V_b = V_c \quad \text{(ii)} \]
\[ V_e = 0.040 \text{ m}^3 \]
\[ T_a = T_c \quad \text{(iii)} \]
\[ V_a = 0.010 \text{ m}^3 \]

\( a, b, c \) how look to see if in any point we are missing one variable in PV = nRT

\[ P_e V_c = nRT_c \rightarrow T_c = 296 \text{ K} = T_a \]

\[ P_a = \frac{nRT_a}{V_a} = 8.0 \times 10^5 \text{ Pa} = P_b \]

\[ T_b = \frac{P_b V_b}{nR} = 1184 \text{ K}. \]

\( a, b, c \) is isobaric:

\[ Q_{ab} = nC_p (T_b - T_a) = 60 \text{ kJ} > 0, \quad \text{given to He gas or enters} \]

\( b, c \) is isochoric:

\[ Q_{bc} = nC_V (T_c - T_b) = -36 \text{ kJ} < 0, \quad \text{leaves the gas} \]

\( c, a \) is isothermal:

\[ Q_{ca} = W_{ca} = nR \cdot T_a \ln \left( \frac{V_a}{V_c} \right) = -11 \text{ kJ} < 0, \quad a \quad a \quad a \]

\( a, b \) is isobaric:

\[ \Delta U_{ab} = nC_V (T_b - T_a) = 36 \text{ kJ} > 0, \quad \text{increased} \]

\( b, c \) is isochoric:

\[ \Delta U_{bc} = Q_{bc} = -36 \text{ kJ} < 0, \quad \text{decreased} \]

\( c, a \) is isothermal:

\[ \Delta U_{ca} = 0. \]